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Linear approximation practice problems

Problem 1 :Let f (x) = ∛x . Find the linear approximation at x = 27. Use the linear approximation to approximate ∛27.2SolutionProblem 2 :Use the linear approximation to find approximate values of(i) (123)2/3(ii) (15)1/4(iii) ∛26 SolutionProblem 3 :Consider the implicit function defined by 3(x2 + y2)2 = 100xy . Use a tangent line approximation at the point (3,1) to estimate the value of y when x = 3.1.SolutionProblem 4 :Finding a local linear approximation at a given point is finding the equation of the tangent line at that point.a) Find the local linear approximation of f(x) = x3 - 2x + 3 at the point where x = 2.b) Use your approximation to estimate f(2.1), f(1.9) and f(1.99).Solution 1) L(27.2) = 3.0074 (approximately)2) i) the approximate value of (123)2/3 is 24.733.ii) the approximate value of (15)1/4 is 1.968.iii) the approximate value of ∛26 is 3.0370.3) 1.14 to two decimals.4) L(2) = 10x - 13i) 8 ii) 6 iii) 6.9 Problem 1 :f(x) = x3 - 5x + 12 and x0 = 2SolutionProblem 2 :g(x) = √(x2 + 9) and x0 = -4SolutionProblem 3 :h(x) = x/(x + 1) and x0 = 1SolutionProblem 4 :Since there were no problems on linear approximation on the second practice prelim, we are including some separately.Consider the function f(x) = e2x.(a) Determine the linearization L(x) of f(x) at the point (0, 1).(b) Use your result in (a) to approximate e0.2.SolutionProblem 5 :Find the linear approximation of f(x) = x sin (πx2) about x = 2. Use the approximation to estimate f(1.99)SolutionProblem 6 :Let f be a differentiable function such that f(3) = 2 and f'(3) = 5. If the tangent line at x = 3 is used to find an approximation to a zero of f , that approximation is(A) 0.4 (B) 0.5 (C) 2.6 (D) 3. 4 (E) 5.5SolutionProblem 7 :The approximate value of y = √(4 + sin x) at x = 0.12 obtained from t he tangent to the graph at x = 0(A) 2 (B) 2.03 (C) 2.06 (D) 2.12 (E) 2.24Solution 1) L(2) = 7x - 42) L(-4) = (1/5) (9-4x)3) L(1) = (1/4)(1+x)4) a) 2x b) 0.45) L(2) = 8 x - 16approximate value of f(1.99) is 0.086) x = 2.6, option C7) 2.03, option B Problem 1 :The radius of a circular plate is measured as 12.65 cm instead of the actual length 12.5 cm. find the following in calculating the area of the circular plate:(i) Absolute error (ii) Relative error (iii) Percentage errorSolutionProblem 2 :A sphere is made of ice having radius 10 cm. Its radius decreases from 10 cm to 9 8 . cm. Find approximations for the following :(i) change in the volume (ii) change in the surface areaSolutionProblem 3 :The time T , taken for a complete oscillation of a single pendulum with length l , is given by the equationT = 2π√(l/g)where g is constant. Find the approximate percentage error in the calculated value of T corresponding to an error of 2 percent in the value of 1.SolutionProblem 4 :Show that the percentage error in the nth root of a number is approximately 1/n times the percentage error in the number.SolutionProblem 5 :Sham was given three of the grades of her classmates. These grades were the three closest values to Sham's grade. If Sham's grade is considered the actual value, what is the mean absolute error of her classmates' grades?Sham's Grade: 96.3Classmate 1 : 95.6Classmate 2 : 94.8Classmate 3 : 95.2SolutionProblem 6 :Find the absolute and relative errors. The actual value is 125.68 mm, and the measured value is 119.66 mm.SolutionProblem 7 :The book's length is 12.5cm, but Via measured only 12.4cm. Find the absolute and relative errors.SolutionProblem 8 :The thermometer measures up to the nearest 2 degrees. The temperature was measured at 38° C. Find the relative Error.Solution 1) i) Absolute error = 0.0225π cm2ii) Relative error = 0.0059 cm2iii) Percentage error = 0.6%2) i) Volume decreases by 80π cm3.ii) Surface area decreases by 16π cm2.3) Percentage error = 1%4) dy/y = (1/n) (1/x) dx5) the Mean Absolute Error is 1.16) the Absolute Value is 6.02mm with a Relative Error of 4.79%.7) the Absolute Value is 0.1cm with a Relative Error of 0.8%.8) Therefore, the Relative Error is 2.63%. Kindly mail your feedback to v4formath@gmail.comWe always appreciate your feedback. ©All rights reserved. onlinemath4all.com Time to practice! This screen has a series of practice problems for linear approximations, so you can develop your skills that we introduced on the preceding screen. As you work through the questions, we'll also illustrate a few important points that we'll use as a starting point at in the next Topic.Note that for every problem, you can—if you wish—immediately view the solution with a single click. This is fine to do if you're feeling stuck, or wish to check a key early-step before you finish your calculation.A word of caution, however: Some students simply read our solutions and think: Yeah, that's exactly what I would have done so I won't bother. Later, when faced with an actual exam question, they discover they didn't actually get the approach "in their hands," and then wish that they had taken this opportunity to practice. As with any new skill, the only way to become fully competent and comfortable is by doing it for yourself; merely watching us demonstrate our skills is of little value. We want you to do well! Only you can do this part, developing the necessary neural connections in your brain and between your brain and your hands.Since there's no penalty for getting something wrong here, go ahead and dive in. And make as many mistakes as you need to — and you need to make some, because that's where real learning happens. (It's where your "learning edge" is!) Our goal is for you to be confident and proficient in solving these types of problems, so that they feel routine to you when you encounter them on exam. We're providing the opportunity to practice so you can get there; we sincerely hope you'll take advantage. So far we've considered a function for which $f = f(x)$: our independent variable has been x, and so the relevant rate of change has been with respect to x: $\left. \dfrac{df}{dx}\right|_\text{at a particular value of x}$. Let's now consider a function that has as its independent variable θ: $g = g(\theta),$ and its associated rate of change with respect to $\theta,$ $\left. \dfrac{dg}{d\theta}\right|_\text{at a particular value of θ}$. You're still doing a practice problem on linear approximations, now just with different notation for the variables. As we saw, the preceding problem illustrates one of the most frequently used approximations used in Physics and other fields: $\sin(d \theta) \approx d \theta$ for small values of $d\theta$ when $d\theta$ is measured in radians. This is more commonly expressed as \[\sin(\theta) \approx \theta \quad \text{for small values of }\theta\] when θ is expressed in radians. Notice that this approximation works only for small values of θ that are close to $\theta=0,$ because (I) at our "base point" of $\theta =0,$ $\sin(0) = 0,$ and (II) that's where the rate $\left. \dfrac{dg}{d\theta}\right|_{\theta = 0} = 1$ is valid. At other locations on the sine curve, the function's value and its rate of change are different. To illustrate, let's consider the same function, $g(\theta) = \sin (\theta),$ but at a different value of $\theta.$ As we'll see, at this new point of interest the function changes half-as-quickly as did was in Problem 2. Let's consider the function $g(\theta) = \sin (\theta)$ again as a practice problem for linear approximations, now at $\theta = \pi$ where the function's rate of change is negative. Let's finally consider the sin function one last time, now at a location where its rate of change is zero. The result of Problem 5 is often initially surprising to students: since $\left. \dfrac{dg}{d\theta}\right|_{\theta = \pi/2} = 0,$ the function's value doesn't change—to first order—if you move a tiny bit away from $x = \dfrac{\pi}{2}$. As usual, the further away from $x = \dfrac{\pi}{2}$ we go, the worse our approximation becomes. But as you can see from the interactive Desmos graph, to first order if you walk a little bit to the right or a little bit to the left from $x = \dfrac{\pi}{2},$ you remain quite close to $y=1.$In the next Topic we'll pull together all of the results we've developed so far, and start to draw some generalizations. Have a question or comment about any of the Problems on this screen? Please use the Forum to post!☕ Buy us a coffee We're working to add more, and would appreciate your help to keep going! 😊We'd appreciate your feedback! 😊 Linear Approximation and DifferentialsLinear Approximation ProblemsLinear Approximation EstimateDifferentials Percent Error ProblemPage 2Implicit Differentiation ProblemsFind dy dx By Implicit DifferentiationCalculus Implicit DifferentiationImplicit Differentiation TutorialImplicit Differentiation PracticeFor the following equation, differentiate implicitly to find dydx\frac{dy}{dx}dxdye(x+y)=sin(x)+cos(y)e^{(x + y)} = \sin{(x)} + \cos{(y)}e(x+y)=sin(x)+cos(y)Implicit DifferentiationHardVideoImplicit Differentiation xyImplicit Differentiation Tangent LineFind dydx\frac{dy}{dx}dxdy and the slope of the tangent line at (−2,1)(-2, 1)(−2,1) for the curve given by2x2−3y3=52x^2 - 3y^3 = 52x2−3y3=5Implicit DifferentiationMediumVideoFind The Derivative ImplicitlyFind dydx\frac{dy}{dx}dxdy and the slope of the tangent line at (0,3) for the curve given byy3+x2y5−x4=27y^3 + x^{2}y^{5} - x^4 = 27y3+x2y5−x4=27Implicit DifferentiationEasyVideoImplicit Differentiation Second DerivativeDetermine the first and second derivatives, dydx\frac{dy}{dx}dxdy and d2ydx2\frac{d^{2}y}{dx^2}dx2d2y for the following equationx2+xy=4x^2 + xy = 4x2+xy=4Implicit DifferentiationHardVideoPage 3limx→3(2x+5)\lim_{x \to 3} (2x + 5)limx→3(2x+5)limx→4x2−16x−4\lim_{x \rightarrow 4} \frac{x^2 - 16}{x - 4}limx→4x−4x2−16limx→9x−3x−9\lim_{x\rightarrow 9} \frac{\sqrt{x} - 3}{x - 9} limx→9x−9x−3limx→−3x2−x+12x+3\lim_{x\rightarrow -3} \frac{x^2 - x + 12}{x + 3}limx→−3x+3x2−x+12limt→1t3−tt2−1\lim_{t \rightarrow 1} \frac{t^3 - t}{t^2 - 1} limt→1t2−1t3−t limh→0(h−5)2−25h\lim_{h\rightarrow 0} \frac{(h-5)^2 - 25}{h} limh→0h(h−5)2−25limx→02−x−2x\lim_{x\rightarrow 0} \frac{\sqrt{2 - x} - \sqrt{2}}{x}limx→0x2−x−2Use the squeeze theorem to prove the following important trigonometric limitlimθ→0sin(θ)θ=1\lim_{\theta\rightarrow 0} \frac{\sin(\theta)}{\theta} = 1limθ→0sin(θ)θ=1limθ→0cos(θ)−1θ\lim_{ \theta\rightarrow 0} \frac{\cos(\theta) - 1}{\theta}limθ→0θcos(θ)−1Let g(x)=|x2+x−6|x−2g(x) = \frac{|x^2 + x - 6|}{x - 2} g(x)=x−2|x2+x−6|Find the limit as x→2+x→2−x→2x\rightarrow 2^{+} x\rightarrow 2^{-} x\rightarrow 2 x→2+x→2−x→2One Sided Limits ExamplesLet f(x)=3x−5 f(x) = \frac{3}{x - 5}f(x)=x−53,Evaluate the limit as x→5−x\rightarrow 5^{-}x→5− and x→5+ x\rightarrow 5^{+}x→5+limx→∞2x−1x+1\lim_{x\rightarrow \infty}\frac{2x - 1}{x + 1} limx→∞x+12x−1limx→∞3x2−5x+1x3−1\lim_{x\rightarrow \infty}\frac{3x^2 - 5x + 1}{x^3 - 1} limx→∞x3−13x2−5x+1Limits Involving Infinitylimx→∞2x+1x2−x\lim_{x\rightarrow \infty} \frac{2x + 1}{\sqrt{x^2 - x}} limx→∞x2−x2x+1Limits Approaching Infinitylim−∞sin1x\lim_{x\rightarrow \infty}\sin\frac{1}{x} lim⁡→∞sinx1Limit as x Approaches Infinitylimx→∞(5x−arctanx)\lim_{x\rightarrow \infty}(\frac{5}{x} - \arctan{x}) limx→∞(x5−arctanx)limx→−∞xx2+1\lim_{x\rightarrow -\infty} \frac{x}{\sqrt{x^2 + 1}}limx→−∞x2+1x limθ→0sec(2θ)tan(30)5θ\lim_{\theta\rightarrow 0} \frac{\sec{(2\theta)} \tan{(3 \theta)}}{5 \theta} limθ→05θsec(2θ)tan(30)Limits With Trig Functionslimx→0tanxx\lim_{x\rightarrow 0} \frac{\tan{x}}{x} limx→0xtanxFinding Limits of Trig Functionslimx→0sin(3x)x\lim_{x\rightarrow 0} \frac{\sin{(3x)}}{x} limx→0xsin(3x)Page 4Use the chain rule to find the derivative of the following function,f(x)=(4x2+5)10f(x) = {(4x^2 + 5)}^{10}f(x)=(4x2+5)10Use the chain rule to find the derivative of the following functionf(x)=3x3+10xf(x) = \sqrt{3x^3 + 10x}f(x)=3x3+10xPractice the chain rule by finding the derivative of the following functiony=3x+4y = \sqrt{3x + 4}y=3x+4Chain Rule Trig FunctionsChain Rule For DerivativesChain Rule Problems With SolutionsFind the derivative of the following functiony=ln(x−1)xπ+1y = \frac{\ln{(x - 1)}}{\sqrt{x^{\pi} + 1}}y=xπ+1ln(x−1)Chain Rule Example ProblemFind the derivative of the following functionf(n)=sin(n2+en+1)f(n) = \sin{(n^2 + e^n + 1)}f(n)=sin(n2+en+1)Page 5Product Rule DifferentiationProduct Rule For ExponentsQuotient Rule DerivativesGiven f(x)f(x)f(x), find the equation of the tangent line at the point (−1,−2)(-1, -2)(−1,−2)f(x)=4x(1+x2)f(x) = \frac{4x}{(1 + x^2)}f(x)=(1+x2)4xProduct and Quotient RuleHardVideo Share — copy and redistribute the material in any medium or format for any purpose, even commercially. Adapt — remix, transform, and build upon the material for any purpose, even commercially. The licensor cannot revoke these freedoms as long as you follow the license terms. Attribution — You must give appropriate credit , provide a link to the license, and indicate if changes were made . You may do so in any reasonable manner, but not in any way that suggests the licensor endorses you or your use. ShareAlike — If you remix, transform, or build upon the material, you must distribute your contributions under the same license as the original. No additional restrictions — You may not apply legal terms or technological measures that legally restrict others from doing anything the license permits. You do not have to comply with the license for elements of the material in the public domain or where your use is permitted by an applicable exception or limitation . No warranties are given. The license may not give you all of the permissions necessary for your intended use. For example, other rights such as publicity, privacy, or moral rights may limit how you use the material. Mobile Notice You appear to be on a device with a "narrow" screen width (i.e. you are probably on a mobile phone). Due to the nature of the mathematics on this site it is best viewed in landscape mode. If your device is not in landscape mode many of the equations will run off the side of your device (you should be able to scroll/swipe to see them) and some of the menu items will be cut off due to the narrow screen width. For problems 1 & 2 find a linear approximation to the function at the given point. Show Mobile Notice Show All Notes Hide All Notes Mobile Notice You appear to be on a device with a "narrow" screen width (i.e. you are probably on a mobile phone). Due to the nature of the mathematics on this site it is best viewed in landscape mode. If your device is not in landscape mode many of the equations will run off the side of your device (you should be able to scroll/swipe to see them) and some of the menu items will be cut off due to the narrow screen width. Here are a set of practice problems for the Calculus I notes. Click on the "Solution" link for each problem to go to the page containing the solution. Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section. Here is a listing of sections for which practice problems have been written as well as a brief description of the material covered in the notes for that particular section. Review - In this chapter we give a brief review of selected topics from Algebra and Trig that are vital to surviving a Calculus course. Included are Functions, Trig Functions, Solving Trig Equations and Equations, Exponential/Logarithm Functions and Solving Exponential/Logarithm Equations. Functions – In this section we will cover function notation/evaluation, determining the domain and range of a function and function composition. Inverse Functions – In this section we will define an inverse function and the notation used for inverse functions. We will also discuss the process for finding an inverse function. Trig Functions – In this section we will give a quick review of trig functions. We will cover the basic notation, relationship between the trig functions, the right triangle definition of the trig functions. We will also cover evaluation of trig functions as well as the unit circle (one of the most important ideas from a trig class!) and how it can be used to evaluate trig functions. Solving Trig Equations – In this section we will discuss how to solve trig equations. The answers to the equations in this section will all be one of the "standard" angles that most students have memorized after a trig class. However, the process used here can be used for any answer regardless of it being one of the standard angles or not. Solving Trig Equations with Calculators, Part I – In this section we will discuss solving trig equations when the answer will (generally) require the use of a calculator (i.e. they aren't one of the standard angles). Note however, the process used here is identical to that for when the answer is one of the standard angles. The only difference is that the answers in here can be a little messy due to the need of a calculator. Included is a brief discussion of inverse trig functions. Solving Trig Equations with Calculators, Part II – In this section we will continue our discussion of solving trig equations when a calculator is needed to get the answer. The equations in this section tend to be a little trickier than the "normal" trig equation and are not always covered in a trig class. Exponential Functions –In this section we will discuss exponential functions. We will cover the basic definition of an exponential function, the natural exponential function, i.e. \({\bf e}^{x}\), as well as the properties and graphs of exponential functions Logarithm Functions – In this section we will discuss logarithm functions, evaluation of logarithms and their properties. We will discuss many of the basic manipulations of logarithms that commonly occur in Calculus (and higher) classes. Included is a discussion of the natural (\(\ln(x)\)) and common logarithm (\(\log(x)\)) as well as the change of base formula. Exponential and Logarithm Equations – In this section we will discuss various methods for solving equations that involve exponential functions or logarithm functions. Common Graphs – In this section we will do a very quick review of many of the most common functions and their graphs that typically show up in a Calculus class. Limits - In this chapter we introduce the concept of limits. We will discuss the interpretation/meaning of a limit, how to evaluate limits, the definition and evaluation of one-sided limits, evaluation of infinite limits, evaluation of limits at infinity, continuity and the Intermediate Value Theorem. We will also give a brief introduction to a precise definition of the limit and how to use it to evaluate limits Tangent Lines and Rates of Change – In this section we will introduce two problems that we will see time and again in this course : Rate of Change of a function and Tangent Lines to functions. Both of these problems will be used to introduce the concept of limits, although we won't formally give the definition or notation until the next section. The Limit – In this section we will introduce the notation of the limit. We will also take a conceptual look at limits and try to get a grasp on just what they are and what they can tell us. We will be estimating the value of limits in this section to help us understand what they tell us. We will actually start computing limits in a couple of sections. One-Sided Limits – In this section we will introduce the concept of one-sided limits. We will discuss the differences between one-sided limits and limits as well as how they are related to each other. Limit Properties – In this section we will discuss the properties of limits that we'll need to use in computing limits (as opposed to estimating them as we've done to this point). We will also compute a couple of basic limits in this section. Computing Limits – In this section we will looks at several types of limits that require some work before we can use the limit properties to compute them. We will also look at computing limits of piecewise functions and use of the Squeeze Theorem to compute some limits. Infinite Limits – In this section we will look at limits that have a value of infinity or negative infinity. We'll also take a brief look at vertical asymptotes. Limits At Infinity, Part I – In this section we will start looking at limits at infinity, i.e. limits in which the variable gets very large in either the positive or negative sense. We will concentrate on polynomials and rational expressions in this section. We'll also take a brief look at horizontal asymptotes. Limits At Infinity, Part II – In this section we will continue covering limits at infinity. We'll be looking at exponentials, logarithms and inverse tangents in this section. Continuity – In this section we will introduce the concept of continuity and how it relates to limits. We will also see the Intermediate Value Theorem in this section and how it can be used to determine if functions have solutions in a given interval. The Definition of the Limit – In this section we will give a precise definition of several of the limits covered in this section. We will work several basic examples illustrating how to use this precise definition to compute a limit. We'll also give a precise definition of continuity.. Derivatives - In this chapter we introduce Derivatives. We cover the standard derivatives formulas including the product rule, quotient rule and chain rule as well as derivatives of polynomials, roots, trig functions, inverse trig functions, hyperbolic functions, exponential functions and logarithm functions. We also cover implicit differentiation, related rates, higher order derivatives and logarithmic differentiation. The Definition of the Derivative – In this section we define the derivative, give various notations for the derivative and work a few problems illustrating how to use the definition of the derivative to actually compute the derivative of a function. Interpretation of the Derivative – In this section we give several of the more important interpretations of the derivative. We discuss the rate of change of a function, the velocity of a moving object and the slope of the tangent line to a graph of a function. Differentiation Formulas – In this section we give most of the general derivative formulas and properties used when taking the derivative of a function. Examples in this section concentrate mostly on polynomials, roots and more generally variables raised to powers. Product and Quotient Rule – In this section we will give two of the more important formulas for differentiating functions. We will discuss the Product Rule and the Quotient Rule allowing us to differentiate functions that, up to this point, we were unable to differentiate. Derivatives of Trig Functions – In this section we will discuss differentiating trig functions. Derivatives of all six trig functions are given and we show the derivation of the derivative of \(\sin(x)\) and \(\tan(x)\). Derivatives of Exponential and Logarithm Functions – In this section we derive the formulas for the derivatives of the exponential and logarithm functions. Derivatives of Inverse Trig Functions – In this section we give the derivatives of all six inverse trig functions. We show the derivation of the formulas for inverse sine, inverse cosine and inverse tangent. Derivatives of Hyperbolic Functions – In this section we define the hyperbolic functions, give the relationships between them and some of the basic facts involving hyperbolic functions. We also give the derivatives of each of the six hyperbolic functions and show the derivation of the formula for hyperbolic sine. Chain Rule – In this section we discuss one of the more useful and important differentiation formulas, The Chain Rule. With the chain rule in hand we will be able to differentiate a much wider variety of functions. As you will see throughout the rest of your Calculus courses a great many of derivatives you take will involve the chain rule! Implicit Differentiation – In this section we will discuss implicit differentiation. Not every function can be explicitly written in terms of the independent variable, e.g. y = f(x) and yet we will still need to know what f'(x) is. Implicit differentiation will allow us to find the derivative in these cases. Knowing implicit differentiation will allow us to do one of the more important applications of derivatives, Related Rates (the next section). Related Rates – In this section we will discuss the only application of derivatives in this section, Related Rates. In related rates problems we are give the rate of change of one quantity in a problem and asked to determine the rate of one (or more) quantities in the problem. This is often one of the more difficult sections for students. We work quite a few problems in this section so hopefully by the end of this section you will get a decent understanding on how these problems work. Higher Order Derivatives – In this section we define the concept of higher order derivatives and give a quick application of the second order derivative and show how implicit differentiation works for higher order derivatives. Logarithmic Differentiation – In this section we will discuss logarithmic differentiation. Logarithmic differentiation gives an alternative method for differentiating products and quotients (sometimes easier than using product and quotient rule). More importantly, however, is the fact that logarithm differentiation allows us to differentiate functions that are in the form of one function raised to another function, i.e. there are variables in both the base and exponent of the function. Applications of Derivatives - In this chapter we will cover many of the major applications of derivatives. Applications included are determining absolute and relative minimum and maximum function values (both with and without constraints), sketching the graph of a function without using a computational aid, determining the Linear Approximation of a function, L'Hospital's Rule (allowing us to compute some limits we could not prior to this), Newton's Method (allowing us to approximate solutions to equations) as well as a few basic Business applications. Rates of Change – In this section we review the main application/interpretation of derivatives from the previous chapter (i.e. rates of change) that we will be using in many of the applications in this chapter. Critical Points – In this section we give the definition of critical points. Critical points will show up in most of the sections in this chapter, so it will be important to understand them and how to find them. We will work a number of examples illustrating how to find them for a wide variety of functions. Minimum and Maximum Values – In this section we define absolute (or global) minimum and maximum values of a function and relative (or local) minimum and maximum values of a function. It is important to understand the difference between the two types of minimum/maximum (collectively called extrema) values for many of the applications in this chapter and so we use a variety of examples to help with this. We also give the Extreme Value Theorem and Fermat's Theorem, both of which are very important in the many of the applications we'll see in this chapter. Finding Absolute Extrema – In this section we discuss how to find the absolute (or global) minimum and maximum values of a function. In other words, we will be finding the largest and smallest values that a function will have. The Shape of a Graph, Part I – In this section we will discuss what the first derivative of a function can tell us about the graph of a function. The first derivative will allow us to identify the relative (or local) minimum and maximum values of a function and where a function will be increasing and decreasing. We will also give the First Derivative test which will allow us to classify critical points as relative minimums, relative maximums or neither a minimum or a maximum. The Shape of a Graph, Part II – In this section we will discuss what the second derivative of a function can tell us about the graph of a function. The second derivative will allow us to determine where the graph of a function is concave up and concave down. The second derivative will also allow us to identify any inflection points (i.e. where concavity changes) that a function may have. We will also give the Second Derivative Test that will give an alternative method for identifying some critical points (but not all) as relative minimums or relative maximums. The Mean Value Theorem – In this section we will give Rolle's Theorem and the Mean Value Theorem. With the Mean Value Theorem we will prove a couple of very nice facts, one of which will be very useful in the next chapter. Optimization Problems – In this section we will be determining the absolute minimum and/or maximum of a function that depends on two variables given some constraint, or relationship, that the two variables must always satisfy. We will discuss several methods for determining the absolute minimum or maximum of the function. Examples in this section tend to center around geometric objects such as squares, boxes, cylinders, etc. More Optimization Problems – In this section we will continue working optimization problems. The examples in this section tend to be a little more involved and will often involve situations that will be more easily described with a sketch as opposed to the 'simple' geometric objects we looked at in the previous section. L'Hospital's Rule and Indeterminate Forms – In this section we will revisit indeterminate forms and limits and take a look at L'Hospital's Rule. L'Hospital's Rule will allow us to evaluate some limits we were not able to previously. Linear Approximations – In this section we discuss using the derivative to compute a linear approximation to a function. We can use the lienar approximation to a function to approximate values of the function at certain points. While it might not seem like a useful thing to do with when we have the function there really are reasons that one might want to do this. We give two ways this can be useful in the examples. Differentials – In this section we will compute the differential for a function. We will give an application of differentials in this section. However, one of the more important uses of differentials will come in the next chapter and unfortunately we will not be able to discuss it until then. Newton's Method – In this section we will discuss Newton's Method. Newton's Method is an application of derivatives that will allow us to approximate solutions to an equation. There are many equations that cannot be solved directly and with this method we can get approximations to the solutions to many of those equations. Business Applications – In this section we will give a cursory discussion of some basic applications of derivatives to the business field. We will revisit finding the maximum and/or minimum function value and we will define the marginal cost function, the average cost, the revenue function, the marginal revenue function and the marginal profit function. Note that this section is only intended to introduce these concepts and not teach you everything about them. Integrals - In this chapter we will give an introduction to definite and indefinite integrals. We will discuss the definition and properties of each type of integral as well as how to compute them including the Substitution Rule. We will give the Fundamental Theorem of Calculus showing the relationship between derivatives and integrals. We will also discuss the Area Problem, an important interpretation of the definite integral. Indefinite Integrals – In this section we will start off the chapter with the definition and properties of indefinite integrals. We will not be computing many indefinite integrals in this section. This section is devoted to simply defining what an indefinite integral is and to give many of the properties of the indefinite integral. Actually computing indefinite integrals will start in the next section. Computing Indefinite Integrals – In this section we will compute some indefinite integrals. The integrals in this section will tend to be those that do not require a lot of manipulation of the function we are integrating in order to actually compute the integral. As we will see starting in the next section many integrals do require some manipulation of the function before we can actually do the integral. We will also take a quick look at an application of indefinite integrals. Substitution Rule for Indefinite Integrals – In this section we will start using one of the more common and useful integration techniques – The Substitution Rule. With the substitution rule we will be able integrate a wider variety of functions. The integrals in this section will all require some manipulation of the function prior to integrating unlike most of the integrals from the previous section where all we really needed were the basic integration formulas. More Substitution Rule – In this section we will continue to look at the substitution rule. The problems in this section will tend to be a little more involved than those in the previous section. Area Problem – In this section we start off with the motivation for definite integrals and give one of the interpretations of definite integrals. We will be approximating the amount of area that lies between a function and the \(x\)-axis. As we will see in the next section this problem will lead us to the definition of the definite integral and will be one of the main interpretations of the definite integral that we'll be looking at in this material. Definition of the Definite Integral – In this section we will formally define the definite integral, give many of its properties and discuss a couple of interpretations of the definite integral. We will also look at the first part of the Fundamental Theorem of Calculus which shows the very close relationship between derivatives and integrals Computing Definite Integrals – In this section we will take a look at the second part of the Fundamental Theorem of Calculus. This will show us how we compute definite integrals without using (the often very unpleasant) definition. The examples in this section can all be done with a basic knowledge of indefinite integrals and will not require the use of the substitution rule. Included in the examples in this section are computing definite integrals of piecewise and absolute value functions. Substitution Rule for Definite Integrals – In this section we will revisit the substitution rule as it applies to definite integrals. The only real requirements to being able to do the examples in this section are being able to do the substitution rule for indefinite integrals and understanding how to compute definite integrals in general. Applications of Integrals - In this chapter we will take a look at some applications of integrals. We will look at Average Function Value, Area Between Curves, Volume (both solids of revolution and other solids) and Work. Average Function Value – In this section we will look at using definite integrals to determine the average value of a function on an interval. We will also give the Mean Value Theorem for Integrals. Area Between Curves – In this section we'll take a look at one of the main applications of definite integrals in this chapter. We will determine the area of the region bounded by two curves. Volumes of Solids of Revolution / Method of Rings – In this section, the first of two sections devoted to finding the volume of a solid of revolution, we will look at the method of rings/disks to find the volume of the object we get by rotating a region bounded by two curves (one of which may be the \(x\) or \(y\)-axis) around a vertical or horizontal axis of rotation. Volumes of Solids of Revolution / Method of Cylinders – In this section, the second of two sections devoted to finding the volume of a solid of revolution, we will look at the method of cylinders/shells to find the volume of the object we get by rotating a region bounded by two curves (one of which may be the \(x\) or \(y\)-axis) around a vertical or horizontal axis of rotation. More Volume Problems – In the previous two sections we looked at solids that could be found by treating them as a solid of revolution. Not all solids can be thought of as solids of revolution and, in fact, not all solids of revolution can be easily dealt with using the methods from the previous two sections. So, in this section we'll take a look at finding the volume of some solids that are either not solids of revolutions or are not easy to do as a solid of revolution. Work – In this section we will look at is determining the amount of work required to move an object subject to a force over a given distance.