Photo: Rhett Allain. Moment of Inertia for a solid sphere.What is the moment of inertia and why would you wan to find it for a sphere? Will this even be fun? The answer to the last question — YES. Trust me, I'm going to make it fun.This is just a quick review. I mean, if you have read this far then I suspect you already have an idea about the moment of inertia. First, let me be clear — there are two different moments of inertia (and they are both represented by I). If you are rotating a rigid object about a FIXED axis then I is a scalar value (that's the one we will calculate here). However, if the object is free to rotate in any direction then I is a tensor. If you are looking for stuff about the tensor version, then this other post is for you.Now back to the scalar version of I. If you have a bunch of masses connected together and rotating about a fixed axis with an angular speed ω, then it's possible to write the rotational kinetic energy as:Where for a finite number of point mass, I would be: Share — copy and redistribute the material in any medium or format for any purpose, even commercially. Adapt — remix, transform, and build upon the material for any purpose, even commercially. The licensor cannot revoke these freedoms as long as you follow the license terms. Attribution — You must give appropriate credit , provide a link to the license, and indicate if changes were made . You may do so in any reasonable manner, but not in any way that suggests the licensor endorses you or your use. ShareAlike — If you remix, transform, or build upon the material, you must distribute your contributions under the same license as the original. No additional restrictions — You may not apply legal terms or technological measures that legally restrict others from doing anything the license permits. You do not have to comply with the license for elements of the material in the public domain or where your use is permitted by an applicable exception or limitation . No warranties are given. The license may not give you all of the permissions necessary for your intended use. For example, other rights such as publicity, privacy, or moral rights may limit how you use the material. Last updated on May 14th, 2022 at 02:08 pm In this post, we will discuss a couple of interesting questions related to Angular Momentum, Torque, moment of inertia & Rotational motion. How does angular momentum work in rotational motion? The angular momentum of a rotating object is equal to the product of its moment of inertia and its angular velocity. If there is no external torque on the object, then its angular momentum remains constant. If there is no external force on an object with linear momentum, then as the object can't change its mass, its velocity also remains constant. But a rotating object can change its moment of inertia, so, even without external torques, its rotational speed can be changed. Does momentum apply to objects that rotate? Just by the term Momentum, we understand linear momentum. Momentum or linear momentum applies to objects moving from one location to another. Since such motion is typically described in straight lines, physicists sometimes call this linear momentum. In rotational motion objects (such as a spinning top) or systems of objects (such as planets in a solar system) rotate around a central point or axis. In rotational motion, we have a separate set of physical quantities that are similar or equivalent to the physical quantities that describe linear motion. Position in linear motion is replaced by angle; velocity is replaced by angular velocity; acceleration is replaced by angular acceleration, and force is replaced by torque. [See this: linear motion & rotational motion equivalents and relationships] The rotational analog to momentum is called angular momentum, and in any system of objects, in the absence of external torques, the angular momentum in the system is conserved. See also ICSE Physics Class 10 Force NumericalsHence, we can say that angular momentum applies to objects that rotate. How does torque take the place of force when measuring rotational motion? All of us should have had a common experience that illustrates how torque works. Suppose we want to push open a door that rotates about its hinges. We know that the speed with which the door opens depends on how hard we push. This push is the force we apply. Speed of opening of the door also depends on how far from the hinges we push—the farther, the faster. It also depends on the angle at which we push. Pushing at a right angle to the door is much more effective than pushing at a smaller or larger angle. If we push at a right angle, then torque equals the force times the distance from the axis of rotation. What takes the place of mass when measuring rotational motion? Mass is defined as the net force on an object divided by its acceleration. By analogy, then, the property that takes the place of mass should be the torque divided by angular acceleration. The property is called rotational inertia or the moment of inertia. It depends not only on mass but on how far the mass is from the axis of rotation. The further the mass is from the axis, the larger the moment of inertia. If we sit on a rotating chair while holding heavy weights, the further we extend our arms, the more becomes the moment of inertia. That means the more difficult it is for someone to start us rotating. That is the concept of inertial in rotation and we call it the moment of inertia. That is, it will require more torque to achieve the same angular acceleration when the moment of inertia is more. Use spherical coordinates to find the moment of inertia about the z-axis of a solid of uniform density bounded by the hemisphere [tex]\rho=cos\varphi[/tex], [tex]\pi/4\leq\varphi\leq\pi/2[/tex], and the cone [tex]\varphi=4[/tex]. Homework Equations [tex]I_{z} = \int\int\int(x^{2}+y^{2})\rho(x, y, z) dV[/tex] The Attempt at a Solution I tried to convert that equation to cylindrical coordinates and got this (k representing density because it's uniform) [tex]I_{z} = k \int^{2\pi}_{0}\int^{\pi/2}_{\pi/4}\int^{1}_{0}\rho^2 sin^{2}\varphi*d\rho*d\varphi*d\theta[/tex] Plugged that into my calculator and got: [tex]\frac{k\pi(\pi+2)}{12}[/tex] The book answer is: [tex]\frac{k\pi}{192}[/tex] What am I doing wrong? LCKurtz Use spherical coordinates to find the moment of inertia about the z-axis of a solid of uniform density bounded by the hemisphere [tex]\rho=cos\varphi[/tex], [tex]\pi/4\leq\varphi\leq\pi/2[/tex], and the cone [tex]\varphi=4[/tex]. You mean [itex]\varphi = \pi / 4[/itex] for the cone. [tex]I_{z} = \int\int\int(x^{2}+y^{2})\rho(x, y, z) dV[/tex] The Attempt at a Solution I tried to convert that equation to cylindrical coordinates and got this (k representing density because it's uniform) [tex]I_{z} = k \int^{2\pi}_{0}\int^{\pi/2}_{\pi/4}\int^{1}_{0}\rho^2 sin^{2}\varphi*d\rho*d\varphi*d\theta[/tex] That doesn't look like cylindrical coordinates to me. But then, why would you want cylindrical coordinates anyway? Assuming that the [itex]\rho^2\sin^2(\phi)[/itex] is the moment arm, check your spherical coordinate dV. Plugged that into my calculator and got: [tex]\frac{k\pi(\pi+2)}{12}[/tex] The book answer is: [tex]\frac{k\pi}{192}[/tex] What am I doing wrong? Aside from getting the dV wrong, using a calculator? [Edit] Looking closer your limits for [itex]\rho[/itex] are also wrong. Last edited: Apr 3, 2010 You mean [itex]\varphi = \pi / 4[/itex] for the cone. Yes, sorry, typo. That doesn't look like cylindrical coordinates to me. But then, why would you want cylindrical coordinates anyway? Assuming that the [itex]\rho^2\sin^2(\phi)[/itex] is the moment arm, check your spherical coordinate dV. Sorry, typo, I meant spherical coordinates. I checked and dV should be [itex]\rho^2\sin^2(\phi)d\rho*d\varphi*d\theta[/itex] So that should change the integral to: [tex]I_{z} = k \int^{2\pi}_{0}\int^{\pi/2}_{\pi/4}\int^{1}_{0}(\rho^2 sin^{2}\varphi)^{2}*d\rho*d\varphi*d\theta[/tex] Aside from getting the dV wrong, using a calculator? I usually use my calculator to check my setup, then once I know that is right, I go back and evaluate it by hand. [Edit] Looking closer your limits for [itex]\rho[/itex] are also wrong. I'm not sure what to do for [itex]\rho[/itex], I thought since the radius of the hemisphere was 1, then [itex]\rho[/itex] would go from 0 to 1. LCKurtz I checked and dV should be [itex]\rho^2\sin^2(\phi)d\rho*d\varphi*d\theta[/itex] The sine should not be squared. So that should change the integral to: [tex]I_{z} = k \int^{2\pi}_{0}\int^{\pi/2}_{\pi/4}\int^{1}_{0}(\rho^2 sin^{2}\varphi)^{2}*d\rho*d\varphi*d\theta[/tex] I'm not sure what to do for [itex]\rho[/itex], I thought since the radius of the hemisphere was 1, then [itex]\rho[/itex] would go from 0 to 1. Have you drawn a picture of the desired volume? Your sphere is not centered at the origin and its equation isn't [itex]\rho[/itex] = 1. Yes, you are correct in assuming that spherical coordinates are similar to polar coordinates, but in three dimensions. They consist of three coordinates: r, θ, and φ, where r is the distance from the origin, θ is the angle from the positive z-axis, and φ is the angle from the positive x-axis in the xy-plane. To derive the moment of inertia for a solid sphere using spherical coordinates, we can use the formula: I = ∫∫∫ (r^2sin^2θ)(ρ)dV where r is the distance from the axis of rotation, θ is the angle from the axis of rotation, ρ is the density of the sphere, and dV is the volume element. First, we need to find the density of the sphere, which is given by ρ = m/V, where m is the mass of the sphere and V is its volume. Since we are dealing with a solid sphere, we can use the formula for the volume of a sphere: V = (4/3)πr^3. Substituting this into the equation for density, we get ρ = (3m)/(4πr^3). Now, we can plug this into the formula for moment of inertia and integrate over the volume of the sphere: I = ∫∫∫ (r^2sin^2θ)(3m)/(4πr^3)dV Next, we need to express the volume element in terms of spherical coordinates. In this case, the volume element is given by dV = r^2sinθdrdθdφ. Substituting this into the integral, we get: I = ∫∫∫ (r^2sin^2θ)(3m)/(4πr^3)(r^2sinθdrdθdφ) Simplifying this, we get: I = (3m)/(4π) ∫∫∫ sin^3θdθdφdr Now, we can evaluate the integral with respect to θ and φ, giving us: I = (3m)/(4π) ∫ (0 to 2π) ∫ (0 to π) sin^3θdθdφ ∫ (0 to R) dr Evaluating the inner integral, we get: I = (3m)/(4π) ∫ (0 to 25992 Students Helped in Physics 101 Mechanics "Numerade has a great goal - to increase people's educational levels all around the world. Educators do not complete student's personal homework tasks. We create video tutorials that may be used for many years in the future." Dr. Mei Lin Chen PhD in Physics 101 Mechanics Welcome to the Robertson Library's open textbook authoring platform and catalogue of open textbooks, part of a comprehensive effort by the University of PEI to deliver quality educational content while reducing the high costs of education. Here you will find original texts authored by staff at the University, including collaborations with colleagues. If you would like more information about how you could author or add your own open texts to the site, contact Keri McCaffrey – knmccaffrey@upei.ca. Homework Statement Derive the formula for the moment of inertia of a thin spherical shell using spherical coordinates and multiple integrals. Relevant Equations Moment of Intertia is (2MR^2)/3 Homework Statement: Derive the formula for the moment of inertia of a thin spherical shell using spherical coordinates and multiple integrals. Homework Equations: Moment of Intertia is (2MR^2)/3 I = (2MR^2)/3 The word "thin" here is important. That is telling you that all the mass is a the same radius. Do you know how to formulate the differential area on the surface of a sphere in spherical coordinates? If so, then you're practically there. Simply assign a symbol for the mass per unit area, express the necessary radius to a typical point, and integrate over the area, and presto! Likes WWGD The word "thin" here is important. That is telling you that all the mass is a the same radius. Do you know how to formulate the differential area on the surface of a sphere in spherical coordinates? If so, then you're practically there. Simply assign a symbol for the mass per unit area, express the necessary radius to a typical point, and integrate over the area, and presto! The problem is I don't know what I'm doing. The professor introduced double integrals, triple integrals, and moment of inertia today and I need to be able to derive the formula by Friday. Last edited: Nov 5, 2019 Do you know what axis is intended for the moment of inertia? They will all be the same for an axis through the center of the sphere, but infinitely many other axes are also possible. Also, you did not answer my question about whether you know how to express the differential area in spherical coordinates. This is a necessary first step. Likes sysprog Do you know what axis is intended for the moment of inertia? They will all be the same for an axis through the center of the sphere, but infinitely many other axes are also possible. Also, you did not answer my question about whether you know how to express the differential area in spherical coordinates. This is a necessary first step. In cartesian coordinates, rotation about the z-axis. I understand how spherical coordinates work but I don't really know how to do integrate in spherical coordinates. The professor introduced moment of inertia, integration in spherical and cylindrical coordinates, and double and triple integrals today, so I am very lost. I understand how spherical coordinates work, and I can integrate, find area, volume, etc. in cartesian coordinates. Likes sysprog I understand how spherical coordinates work So define a surface element in those terms and write an expression for its moment of inertia about the axis. Post however far you get. Likes sysprog I think calculus class sometimes spends all the time teaching students how to solve integrals and loses track of why you are doing integrals, what the integral means, or how a problem becomes an integral problem. Here's the idea: you know the answer for a small piece, and you are trying to write the answer for the whole thing as a sum of all the small pieces. Do you know what the inertia is for some small piece? Can you describe a thin shell as collection of those small pieces? Can you make the description of the location of each and every piece parametric in the coordinates? Then the inertia of the shell is the inertia of those small pieces summed over the whole shell. Taking the pieces as infinitesimally small, the sum becomes an integral. Yes, you will have to integrate in three dimensions. Likes Delta2, jake010 and sysprog Yes, you will have to integrate in three dimensions. Actually, there is only the need for a 2D integration. This is where the key word "thin" comes into play. There is no need to integrate radially. Likes Delta2 and sysprog jtbell The professor introduced moment of inertia, integration in spherical and cylindrical coordinates, and double and triple integrals today Have you done any surface integrals in Cartesian coordinates, where the surface element is ##dx\,dy##? Likes sysprog Actually, there is only the need for a 2D integration. This is where the key word "thin" comes into play. There is no need to integrate radially. One of the integrals is trivial, but does result in multiplying by the thickness. We are used to short cutting the third integral, but the third dimension is required. Likes sysprog One of the integrals is trivial, but does result in multiplying by the thickness. We are used to short cutting the third integral, but the third dimension is required. Yes, I agree that most spheres are three dimensional. However, the key word "thin" tells us that the intent was to assume a two dimensional surface for the sphere, thus only requiring two integrations. Likes sysprog Yes, I agree that most spheres are three dimensional. However, the key word "thin" tells us that the intent was to assume a two dimensional surface for the sphere, thus only requiring two integrations. If you don't integrate in the third dimension you get the wrong answer. We know the integrand doesn't vary significantly in the radial dimension. We know the radial integral is trivial. So we don't write down the third integral but we DO multiply by the thickness. That thickness is the solution to the third integral. When you multiply by the thickness, guess what, you just integrated the third dimension. Otherwise where did that thickness come from? Just because we are so used to it that we skip steps and jump directly to the answer doesn't mean all three dimensions are not required. If you tell this poster to only consider two dimensions he will not have the thickness in the answer. The only way he is going to have it in the answer is if he writes and solves the third integral. Likes jake010 and sysprog If you don't integrate in the third dimension you get the wrong answer. We know the integrand doesn't vary significantly in the radial dimension. We know the radial integral is trivial. So we don't write down the third integral but we DO multiply by the thickness. That thickness is the solution to the third integral. When you multiply by the thickness, guess what, you just integrated the third dimension. Otherwise where did that thickness come from? Just because we are so used to it that we skip steps and jump directly to the answer doesn't mean all three dimensions are not required. If you tell this poster to only consider two dimensions he will not have the thickness in the answer. The only way he is going to have it in the answer is if he writes and solves the third integral. This is as far as I've gotten: dI of shell = dI solid sphere/dR dI of solid sphere $=r^2dm = \rho r^2dv$ dv=(rdφ)(dr)(rsinφdθ) $dI=\rho r^2r^2$sinφdrdθdφ I integrate with respect to r from 0 to R, with respect to theta from 0 to 2π, and with respect to φ from 0 to π , then take a derivative with respect to R, and get the wrong answer. What am I doing wrong? If you tell this poster to only consider two dimensions he will not have the thickness in the answer. The only way he is going to have it in the answer is if he writes and solves the third integral. He will, if he knows that "thin" means. But whatever, ... (this is really boring at this point). I integrate with respect to r from 0 to R, with respect to theta from 0 to 2π, and with respect to φ from 0 to π and get the wrong answer. What am I doing wrong? Why would you integrated from 0 to R? Have you thought about what "thin" means? Have you drawn a good picture, showing the whole situation in a perspective drawing? If not, it would help a lot. There is more to this than plug and chug. Likes jake010 I know what thin means which is why I'm taking a derivative with respect to R. I don't think you are really taking any derivatives at all, but if you think so, ... I do not seem to be getting through to you, so I'm done here. I'm sure someone else will help you. I don't think you are really taking any derivatives at all, but if you think so, ... I do not seem to be getting through to you, so I'm done here. I'm sure someone else will help you. You're not getting through to me because I need detailed explanations of what I need to do. I've made it very clear from the start that I just learned about moment of inertia and triple integrals yesterday and that I'm very lost and need serious assistance. I'm doing my best and if you can't help me please don't bother to reply. In regards to derivatives, this video shows how the moment of inertia for a spherical shell can be found using that of a solid sphere, I understand how what is shown in the video works, so as of now I'm trying to figure out how to find the moment of inertia for a solid sphere. If you know the moment of inertia of a solid sphere you can indeed use that to find the moment of inertia of a thin spherical shell. No integrals are required. Unfortunately, your problem expressly asks you to use multiple integrals, so I'm afraid that isn't going to be acceptable. I think this is a good thing. In making you add up the pieces for one particular shape they are trying to give you an understanding of how the moment of inertia for any shape comes about. So, again, integrating is adding up. Can you say what the moment of inertia is for a very small piece of the sphere? Likes jake010 and sysprog If you know the moment of inertia of a solid sphere you can indeed use that to find the moment of inertia of a thin spherical shell. No integrals are required. Unfortunately, your problem expressly asks you to use multiple integrals, so I'm afraid that isn't going to be acceptable. I think this is a good thing. In making you add up the pieces for one particular shape they are trying to give you an understanding of how the moment of inertia for any shape comes about. So, again, integrating is adding up. Can you say what the moment of inertia is for a very small piece of the sphere? Thanks for the help, I think I'm understanding this better, I got $dI=\rho r^2r^2$sinφdrdθdφ for a small piece of the solid sphere's moment of inertia but I don't think it's correct because the integral doesn't work out correctly. If you know the moment of inertia of a solid sphere you can indeed use that to find the moment of inertia of a thin spherical shell. No integrals are required. Unfortunately, your problem expressly asks you to use multiple integrals, so I'm afraid that isn't going to be acceptable. I think this is a good thing. In making you add up the pieces for one particular shape they are trying to give you an understanding of how the moment of inertia for any shape comes about. So, again, integrating is adding up. Can you say what the moment of inertia is for a very small piece of the sphere? Correct me if I'm wrong but is dI actually equal to ρ((rsinφ)^2)(r^2)sinφdrdθdφ because the radius squared in dI=r^2dm is actually (rsinφ)^2 because we're looking for the component of radius parallel to the xy plane?Because the integral works out to the correct answer when dI is that way. Correct me if I'm wrong but is dI actually equal to ρ((rsinφ)^2)(r^2)sinφdrdθdφ because the radius squared in dI=r^2dm is actually (rsinφ)^2 because we're looking for the component of radius parallel to the xy plane?Because the integral works out to the correct answer when dI is that way. Yes, you are getting it. I'm a little worried that "because the integral works out to the correct answer" is usually not an approach conducive to understanding. So if you don't mind I'd like to coach you a little through the "why" so you can be a little more confident in the future. Before I do, just so I'm not accused of giving anything away, could you tell me the expression for the rotational inertia of a point mass? collinsmark If you don't integrate in the third dimension you get the wrong answer. This isn't true for a problem such as this. (I.e., infinitesimally "thin," hollow shell.) All that's necessary is to define surface density. [itex] \rho_s = \frac{M}{A} [/itex] You can think of it as mass per unit area. (If it helps to think of it in familiar units, perhaps think of it as [itex] \left[\frac{\mathrm{kg}}{\mathrm{m^2}} \right] [/itex].) For this problem, the density of an infinitesimally thin, spherical shell is [itex] \rho_s = \frac{M}{\mathrm{Surface \ area \ of \ a \ sphere}} [/itex]. This shouldn't be too unfamiliar to you. You've probably done this before for linear density. For example, if you were finding the mass per unit length of a thin rod, you would start with [itex] \lambda = \frac{\mathrm{mass \ of \ the \ rod}}{\mathrm{length \ of \ the \ rod}} [/itex]. It's the same idea here with [itex] \rho_s [/itex], except two dimensions are involved instead of just one. The differential mass, [itex] dm [/itex] is calculated by [itex] dm = \rho_s (\mathrm{differential \ length})(\mathrm{differential \ width}) [/itex] A picture might help here. The following is a picture of a small, differential patch of surface on a sphere of radius [itex] r [/itex], where [itex] \theta [/itex] represents longitude and [itex] \varphi [/itex] represents latitude (well, lattidue with 0 deg and 180 deg defining the poles). This differential mass can be used in calculating the moment of inertia, keeping care to properly specify the distance of [itex] dm [/itex] to the axis of rotation. The integral can then be carried out integrating over [itex] \theta [/itex] and [itex] \varphi [/itex] . Only two integrals are necessary. No integration over the radius of the sphere [itex] r [/itex] is necessary. The radius of the sphere treated as a constant. (Don't confuse the radius of the sphere [itex] r [/itex] with the distance from [itex] dm [/itex] to the axis of rotation. They are different entities.) Last edited: Nov 6, 2019 Likes Delta2 and WWGD This is as far as I've gotten: dI of shell = dI solid sphere/dR dI of solid sphere $=r^2dm = \rho r^2dv$ dv=(rdφ)(dr)(rsinφdθ) $dI=\rho r^2r^2$sinφdrdθdφ I integrate with respect to r from 0 to R, with respect to theta from 0 to 2π, and with respect to φ from 0 to π , then take a derivative with respect to R, and get the wrong answer. What am I doing wrong? Oops! My bad! In the previous rapid exchanges I missed the above quoted post. Yes, along with your last post fixing the r^2 you have it completely. Now let me explain why the r^2 is wrong. You have two different r's. You have r as in the radial dimension in spherical coordinates. But the r in the expression for rotational inertia mr^2 is not that r. It is the radial distance from the rotational axis. Likes Delta2 This isn't true for a problem such as this. (I.e., infinitesimally "thin," hollow shell.) All that's necessary is to define surface density. [itex] \rho_s = \frac{M}{A} [/itex] You can think of it as mass per unit area. (If it helps to think of it in familiar units, perhaps think of it as [itex] \left[\frac{\mathrm{kg}}{\mathrm{m^2}} \right] [/itex].) For this problem, the density of an infinitesimally thin, spherical shell is [itex] \rho_s = \frac{M}{\mathrm{Surface \ area \ of \ a \ sphere}} [/itex]. This shouldn't be too unfamiliar to you. You've probably done this before for linear density. For example, if you were finding the mass per unit length of a thin rod, you would start with [itex] \lambda = \frac{\mathrm{mass \ of \ the \ rod}}{\mathrm{length \ of \ the \ rod}} [/itex]. It's the same idea here with [itex] \rho_s [/itex], except two dimensions are involved instead of just one. The differential mass, [itex] dm [/itex] is calculated by [itex] dm = \rho_s (\mathrm{differential \ length})(\mathrm{differential \ width}) [/itex] A picture might help here. The following is a picture of a small, differential patch of surface on a sphere of radius [itex] r [/itex], where [itex] \theta [/itex] represents longitude and [itex] \varphi [/itex] represents latitude (well, lattidue with 0 deg and 180 deg defining the poles). View attachment 252482 This differential mass can be used in calculating the moment of inertia, keeping care to properly specify the distance of [itex] dm [/itex] to the axis of rotation. The integral can then be carried out integrating over [itex] \theta [/itex] and [itex] \varphi [/itex] . Only two integrals are necessary. No integration over the radius of the sphere [itex] r [/itex] is necessary. The radius of the sphere treated as a constant. (Don't confuse the radius of the sphere [itex] r [/itex] with the distance from [itex] dm [/itex] to the axis of rotation. They are different entities.) It is thin but it is not infinitesimally thin. It is a real object. It is thin enough that the integrand does not change much over the thickness making the integral in the third dimension trivial, but the third dimension must be included. As Jake discovered " dI of solid sphere $=r^2dm = \rho r^2dv$ dv=(rdφ)(dr)(rsinφdθ) $dI=\rho r^2r^2$sinφdrdθdφ " Except the first r is the radial distance from the axis and should be r sin(phi) that is the correct reasoning and the correct integrand with the correct infinitesimal piece of volume. Now answer me this: does the correct moment of inertia for a thin sphere have "dr" in the expression, or does it have thickness? It has thickness. Why? Because that is what dr INTEGRATES to! Yes, you absolutely DO have to integrate in all three dimensions. The fact that the sphere is thin means that the radial integral is trivial, but it is there \int_r1^r2 dr\ = r2-r1 = thickness Likes jake010 collinsmark It is thin but it is not infinitesimally thin. It is a real object. It is thin enough that the integrand does not change much over the thickness making the integral in the third dimension trivial, but the third dimension must be included. As Jake discovered " dI of solid sphere $=r^2dm = \rho r^2dv$ dv=(rdφ)(dr)(rsinφdθ) $dI=\rho r^2r^2$sinφdrdθdφ " Except the first r is the radial distance from the axis and should be r sin(phi) that is the correct reasoning and the correct integrand with the correct infinitesimal piece of volume. Now answer me this: does the correct moment of inertia for a thin sphere have "dr" in the expression, or does it have thickness? It has thickness. Why? Because that is what dr INTEGRATES to! Yes, you absolutely DO have to integrate in all three dimensions. The fact that the sphere is thin means that the radial integral is trivial, but it is there \int_r1^r2 dr\ = r2-r1 = thickness If you are deriving the formula for a spherical shell -- not a thin spherical shell, but a spherical shell that has an inner radius [itex] a [/itex] and an outer radius [itex] b [/itex] -- then yes you would integrate over [itex] r [/itex] from [itex] a [/itex] to [itex] b [/itex]. The moment of intertia would not be a function of a [itex] r [/itex] but instead would be a function of [itex] a [/itex] and [itex] b [/itex]. In that case a triple integral would be involved. But the problem statement specifically stated "thin" spherical shell, implying that the thickenss between [itex] a [/itex] and [itex] b [/itex] is negligible. In such a case, the moment of inertia is a function of the radius [itex] r [/itex]. Only a double integral is necessary. No [itex] dr [/itex] is involved. Likes jake010 and Dr.D @Cutter Ketch wants to insist that this problem be treated as a thick shell. He says that this leads to a trivial integration on the radius, but here he is mistaken. If the thick shell problem is properly formulated, the radial distance from the reference axis to the mass element will vary with the radial position of the mass element. Thus the radial integration will not be the simple integration of dr to get the thickness, but something more complicated. The inclusion of the word "thin" in the problem statement was intended to be significant simplifcation; it is ignored at the expense of considerably more work for no gain. Likes collinsmark This isn't true for a problem such as this. (I.e., infinitesimally "thin," hollow shell.) All that's necessary is to define surface density. [itex] \rho_s = \frac{M}{A} [/itex] You can think of it as mass per unit area. (If it helps to think of it in familiar units, perhaps think of it as [itex] \left[\frac{\mathrm{kg}}{\mathrm{m^2}} \right] [/itex].) For this problem, the density of an infinitesimally thin, spherical shell is [itex] \rho_s = \frac{M}{\mathrm{Surface \ area \ of \ a \ sphere}} [/itex]. This shouldn't be too unfamiliar to you. You've probably done this before for linear density. For example, if you were finding the mass per unit length of a thin rod, you would start with [itex] \lambda = \frac{\mathrm{mass \ of \ the \ rod}}{\mathrm{length \ of \ the \ rod}} [/itex]. It's the same idea here with [itex] \rho_s [/itex], except two dimensions are involved instead of just one. The differential mass, [itex] dm [/itex] is calculated by [itex] dm = \rho_s (\mathrm{differential \ length})(\mathrm{differential \ width}) [/itex] A picture might help here. The following is a picture of a small, differential patch of surface on a sphere of radius [itex] r [/itex], where [itex] \theta [/itex] represents longitude and [itex] \varphi [/itex] represents latitude (well, lattidue with 0 deg and 180 deg defining the poles). View attachment 252482 This differential mass can be used in calculating the moment of inertia, keeping care to properly specify the distance of [itex] dm [/itex] to the axis of rotation. The integral can then be carried out integrating over [itex] \theta [/itex] and [itex] \varphi [/itex] . Only two integrals are necessary. No integration over the radius of the sphere [itex] r [/itex] is necessary. The radius of the sphere treated as a constant. (Don't confuse the radius of the sphere [itex] r [/itex] with the distance from [itex] dm [/itex] to the axis of rotation. They are different entities.) Got it. I understand it now. Sorry I haven't been responding, I was studying on my own. Thank you all so so much for the help! I finally get it. I really appreciate you guys taking the time to help me! Likes collinsmark This online science and technology portal primarily covers High School Physics for K12, ICSE, CBSE, ISC, IGCSE, GCSE, AP, etc. for free Online Physics Learning and reading. Along with High school physics, we also cover a few more important subjects for high school and Engineering (tech) students. This portal covers the following: High School Science: Online Physics Tutorials | Class Notes | Numerical problems & Worksheets | Physics Assignments | Chemistry tutorials | Biology Tutorials, Question/Answer Class 9 Physics (chapter and topic wise) Study material Engineering & Tech: Electronics – Analog, Digital for engineering | Microprocessor tutorials (8085) for engineering | Programming – Java | Software Engineering | Process Control & Instrumentation Physics Notes Categories (chapter-wise) The links below take you to chapter-wise physics notes & topics pages. These hubs or pages contain links to numerous related posts and class notes for your reference. (for K12, ICSE, CBSE, ISC, IGCSE, GCSE, AP, SAT, NEET, JEE, WBJEE) Physics Chapter-wise Links (click the links to get the list of posts on specific chapters) Popular Articles & Notes Physics Numericals || Physics PDF Notes Physics Calculators || Dimensional Formulas Other related subjects we cover Chemistry || Biology Digital Electronics || Microprocessor, & microcontroller Java Programming This High School Physics online Portal was founded by Anupam M. It is a study platform or portal for online learning, with the following features:(1) free reading material (2) High School Course notes (3) lab course material (4) study helps in different formats (5) online calculator The expression for the moment of inertia of a sphere can be developed by summing the moments of infintesmally thin disks about the z axis. The moment of inertia of a thin disk is Show more detail. Mass moment of inertia, or inertia as it will be referred to from here on, is resistance to rotation. The bigger the inertia, the slower the rotation. [latex]\sum M = I\alpha[/latex]. Inertia is always positive and has units of kgm2 or slugft2. For an infinitesimal unit of mass, the inertia depends on how far it is from the axis of rotation. $$ I = \int_m r^2dm $$ As shown in this image, each little dm at r distance from the axis of rotation (y) is added up (through integration). If r is bigger, the inertia is bigger. If there is more mass closer to the axis of rotation, the inertia is smaller. A skill that you can develop is your visualization of the rotation about each axis. As shown in the following figure, rotating about the different axes will produce different types of rotation. You can imagine sticking your pencil into an object and twisting along that axis. In this image, rotation about the y axis and x axis produce different types of rotation. Due to the symmetry, rotation about the x axis and z axis looks identical. The red r's in this image show the distance that is being measured when adding up each little infinitesimal dm. Notice how the r changees direction from x to y but looks the same between x and z. Equations have been developed for common shapes so that you don't have to integrate every time you want to find the inertia of an object. The result is different for each axis, as shown in the following figure. 'Ixx' can be read as 'the inertia if rotating about the x-axis'. Notice for Ixx and Izz that the height and radius of the cylinder affect the inertia, whereas for Iyy, only the radius is considered. The equations for each of the objects are listed in a table below. First is a second explanation of inertia. We start by constructing, in our minds, an idealized object for which the mass is all concentrated at a single location which is not on the axis of rotation: Imagine a massless disk rotating about an axis through the center of the disk and perpendicular to its faces. Let there be a particle of mass m embedded in the disk at a distance r from the axis of rotation. Here's what it looks like from a viewpoint on the axis of rotation, some distance away from the disk: where the axis of rotation is marked with an O. Because the disk is massless, we call the moment of inertia of the construction, the moment of inertia of a particle, with respect to rotation about an axis from which the particle is a distance r. I = mr2 is our equation for the moment of inertia of a particle of mass m, with respect to an axis of rotation from which the particle is a distance r. Now suppose we have two particles embedded in our massless disk, one of mass m1 at a distance r1 from the axis of rotation and another of mass m2 at a distance r2 from the axis of rotation. The moment of inertia of the first one by itself would be I1 = m1r12 and the moment of inertia of the second particle by itself would be I2 = m2r22 The total moment of inertia of the two particles embedded in the massless disk is simply the sum of the two individual moments of inertial. I = I1 + I2 I = m1r12 + m2r22 This concept can be extended to include any number of particles. For each additional particle, one simply includes another miri2 term in the sum where mi is the mass of the additional particle and ri is the distance that the additional particle is from the axis of rotation. In the case of a rigid object, we subdivide the object up into an infinite set of infinitesimal mass elements dm. Each mass element contributes an amount of moment of inertia dI = r2dm to the moment of inertia of the object, where r is the distance that the particular mass element is from the axis of rotation. Source: Calculus-Based Physics 1, Jeffery W. Schnick. Specific inertia equations depending on the shape of the object and axis of rotation can be found below. Notice some of the shapes have multiple sets of axes: [latex]I_{xx} \text{ and } I_{xx}^\prime[/latex]. There are multiple equations. Symmetric Shapes Thin Ring $$ I_{xx} = \frac{1}{2}mr^2 \\I_{yy}=mr^2 \\I_{zz} = \frac{1}{2}mr^2 $$ * thickness